READING IN CONTEXT: THE RECEPTION OF
GEROLAMO CARDANO'S *LIBER DE LUDO ALEAE*

A RESEARCH PAPER

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TO DR. SLOAN DESPEAUX

DEPARTMENT OF MATHEMATICS

WESTERN CAROLINA UNIVERSITY

BY

NATHAN BOWMAN

Any person who enjoys gambling games would immensely desire to master the theory of probability and find a fool-proof way of winning at these games. Many have made valiant attempts at creating a successful system. One man, in the 16ᵗʰ century, genuinely believed that he had cracked the code. This man was Gerolamo Cardano. He detailed his discovery in his book *Liber de Ludo Aleae*, also known as *The Book on Games of Chance.* This book, a discussion of probability and gambling, was frowned upon due to numerous reasons, both moral and practical. Historians of mathematics have often discussed the work by Pascal, Fermat and Bernoulli; however, it seems that Cardano's *Liber de Ludo Aleae* has often been left on the back of the shelf to collect dust. This poses the question of why Cardano's book has been underappreciated. In this paper, we will seek to understand the reception of *Liber de Ludo Aleae* by looking deeper into Cardano as a gambler and mathematician, considering how he perceived his own book, and finally, by evaluating contemporary opinions and scholarly critiques.

Much of what we know about Cardano comes from his own personal writings.[1] He wrote many works including his autobiography entitled *Da Propia Vita.* In it, Cardano wrote that he was born in Pavia on September 24, 1501. He was born outside of wedlock, a situation usually disdained by the general public of the time. Consequently, Cardano's parents attempted to terminate the pregnancy early on; however, their attempts failed. Cardano was left hidden in the countryside while his parents resided separately in Milan. After many years in the country, his parents brought him to Milan where they considered themselves married, and they all lived together. Cardano adored his father, although his father abused him regularly. His father wanted him to pursue a career in the

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[1] Markus Fierz, *Girolamo Cardano (1501-1576): Physician, Natural Philosopher, Mathematician, Astrologer, and Interpreter of Dreams* (Boston: Birkhäuser, 1983), 1.

legal profession, but Cardano decided to enter into the medical field to become a physician. His father did not agree with his decision and thought that Cardano was not rising to his utmost potential. Despite this discouragement, in 1526, at the University of Pavia, Cardano received his degree to become a medical doctor.[2]

Cardano lived an impoverished life. Throughout his poverty, he used the little money he had in dice games and soon became addicted to gambling.[3] Cardano stated, "It was not a love of gambling, but the odium of my estate and a desire to escape, which compelled me."[4]

Throughout his addiction to gambling, Cardano tried to find a job that he deserved. Many employers rejected him, not for his addiction, but for the fact of his illegitimate birth. However, he did not give up hope. Cardano kept researching probability and chance. His gambling addiction led him to discover one of the fundamental laws of the theory of probability. He was possibly the first to realize that there is in fact a theory of chance.[5] His realization spawned an immense amount of curiosity for future research from other mathematicians.

In 1534 Cardano took on the position of physician in Milan's poorhouse as well as a position in the Mathematics department at the University of Milan. He worked at the two jobs until he was offered a professorship of mathematics at the University of Pavia, however this professorship was offered with a great cost. He would first have to endure a probationary period there as well as lower pay. The university might have enacted these

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[2] *Ibid.*, 4
[3] *Ibid.*, 5
[4] Schwartz, David, *Roll The Bones: The History of Gambling* (New York: Gotham Books, 2006), 78.
[5] Fierz, 5.

conditions of employment because of worries about Cardano's gambling addiction or

even his illegitimate birth. Cardano felt that it would be below his dignity to accept this

job. His pride overcame the job opportunity. Although he declined the professorship,

Cardano continued in his research. He later published two books on fate and astrology.

Subsequently, Cardano's first popular book, *Arithmetica,* was published.[6] At this stage,

Cardano had finally made a name for himself. It was now time for him to put all his

findings of probability and gambling together in a book. This text aimed to set out a

theory of probability, as well as to help gamblers around the world succeed in their

gaming endeavors. Many theories have arisen over whether he wrote his book on chance

to justify his addiction, or if he wrote it out of his pure mathematical curiosity. In his

book *Games, Gods, and Gambling*, F.N. David states that he disagrees with other

mathematicians who think it was Cardano's mathematical curiosity that drove him to

research gambling. He believed that it was in fact Cardano's addiction that the book

sought to rationalize.[7]

One of the first works ever written on the theory of probability is Cardano's *Liber*

de Ludo Aleae.[8] It was the first thorough treatment of sample spaces and probability.

Cardano's book is mainly written about dice games and how the die could fall during a

dicing game. There are few scenarios that deal with playing cards. His book showed the

different number of ways that a specified number of dice may fall in a certain dice game.

Cardano worked to formulate the probability of an event as a ratio of favorable to total

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[6] *Ibid.*, 5.

[7] David, F.N., *Games, Gods, and Gambling* (New York: Hafner Publishing Company, 1962), 58-60.

[8] Ore, Oystein, *Cardano: The Gambling Scholar* (Princeton: Princeton University Press, 1953), 143.

causes (or, outcomes).[9] In his book, Cardano began by giving explanations of

terminology and ideas before he delved into his research. He created the mathematical

terms "circuit" and "equality." Modern mathematicians and researchers do not always

appreciate Cardano's book. Cardano called what we think of today as the sample space

as "circuit," and the "equality" referred to the sample space halved.[10] The circuit and

equality being true depends on the state of play and whether or not the game is honest or

rigged. After Cardano discussed this terminology and all the basics that one should know

to interpret his research, he explained to the reader the two methods he used throughout

the book, the standard method and the reasoning on the mean (ROTM) method. His

methods are not clearly stated in *Liber de Ludo Aleae*, however many commentators have

deciphered what these two methods are.[11]

The standard method is what we use today in statistics. It is the most straight-

forward and correct of the two methods. The standard method takes favorable cases and

divides them by the circuit.[12] This is normally the method that is taught in many

introductory statistics classes. For example, if one is casting a single die, the probability

of one throwing an odd number is $^1/_2$. This is understood by dividing the possible odd

number outcomes by the number of total outcomes. By doing this, one would get 3/6,

which simplifies to $^1/_2$.

The reasoning on the mean method (ROTM) is displayed both by itself and

alongside of the standard method in *Liber de Ludo Aleae*. Cardano reasoned in his book

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[9] Williams, Lambert, "Cardano and the Gambler's Habitus," *Studies of History and Philosophy of Science* 36 (2005): 26.

[10] *Ibid*., 27.

[11] Ore, 145.

[12] Lambert, 28.

that in six throws, theoretically each of the faces should have appeared once. When a die

is thrown, each of its six faces should, in the long run, appear equally often.[13] An

example of this in his book is when he discusses the probabilities of repetition of an event

yielded three different solutions. Cardano insisted that each time was right until he

reached another answer. It was not until he reached his third result that he achieved the

correct answer.[14] This concept can potentially be difficult to wrap the mind around.

Additionally, many of the results that he tried to prove with ROTM are erroneous. The

errors show many contradictions between ROTM and the standard method.[15] Cardano

makes it very clear about who, when, where and with whom one should gamble in his

Liber de Ludo Aleae. Along with the two methods that he uses in the book, he also

writes about morals and ethics on who, when and with whom one should gamble.

He begins with the conditions of play. For example, money must be spent in

moderation. Cardano makes it very clear that one must in no circumstances gamble

alone. This could create suspicions that the gambler is developing an addiction.

Gambling is not looked down upon if boys, young men and soldiers participate. In his

opinion, women should not be permitted to gamble at any time because in that time in

history, it would have not been "lady-like" in the least. One's opponent should be on the

same proficiency level as the player to save the humiliation of losing. Cardano calls it a

disgrace to play with an experienced gambler when the player is not as good as the

professional.[16]

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[13] Ore, 140.

[14] *Ibid.,* 145

[15] *Ibid*., 152-153.

[16] Cardano, Gerolamo, *Liber de Ludo Aleae (The Book on Games of Chance)*(Princeton: Princeton University Press, 1953), 3.

There were many reasons as to why Cardano believed that a man should gamble. He referred to these reasons in his chapter "Utility of Play, and Losses."[17] For Cardano, it is permissible to gamble in times of severe grief.[18] As a test of patience, Cardano says that the best idea for gambling is to not gamble at all. When Cardano summed up all these reasons for who, where, when and with whom one should gamble, he referred to the term "The Fundamental Principle of Gambling."[19] The main factors of the Fundamental Principle of Gambling are the equal conditions of: opponents, bystanders, money, situation, and those factors that correspond with honest and dishonest games, the dice box and the die itself.

Honesty is a significant ethical factor in gambling. In his chapter about the hanging dice box and dishonest dice, he alerted all gamblers to watch out for one trying to rig the table and cheat players out of their fair chance of winning. On a round gaming board, the incline of the table is critical to the game. If the table is leaning down towards your opponent, then it will be detrimental to the player's chances. This is opposite for when the table, also known as the dice box, is tilted in your favor. The best scenario occurs when the dice box is completely level in order to give each player a fair chance of winning.[20] Lighting also plays a role in whether or not the game is rigged. He goes into great detail about how the lighting on the table can avert attention to a certain place that would draw one's attention away from the game. The die itself can even be shaved on a

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[17] *Ibid.*, 4.
[18] Cardano, Gerolamo, *Liber de Ludo Aleae (The Book on Games of Chance)* (Princeton: Princeton University Press, 1953), 1-2. "But in times of great anxiety and grief, it is considered to be not only allowable, but even beneficial."
[19] *Ibid.*, 5-6.
[20] *Ibid.*, 7.

certain side so it will always roll in favor of a certain number.[21] From an unwritten code,

once the stakes have been made, the money no longer belongs to anyone until the round

has been completed and the money has been won. So, hypothetically, if the game was to

come to an abrupt stop, then the money would be distributed according to each player's

chance of winning.[22] Perhaps this was a kind of motivation for Cardano to begin to

research probability.

Cardano saw his book as a gambler's manual instead of a mathematical

composition.[23] This is where mathematicians begin to develop a bad taste for the book.

Many problems have risen from the *Liber de Ludo Aleae*.

During their research, modern mathematicians claim to dislike Cardano's book

for various reasons. Some believe that it is just completely wrong, while others just think

that it would have been written better if Cardano would have intended for the book to be

a mathematical text instead of a manual for the common gambler. Thus, for many of

Cardano's contemporaries, the book tended to collect dust on the back of library shelves.

The first main issue that mathematicians have is that *Liber de Ludo Aleae* is a

collaboration of many contradictory statements and results that were put together into a

Renaissance text over an approximate 40 year time span.[24] His audience is also unclear

because we as mathematicians understand this to be a mathematical text, while Cardano

did not write it with that intent. The questions that he presented in his book are not

[21] *Ibid.*, 7

[22] Cooke, Roger, *The History of Mathematics: A Brief Course* (New York: John Wiley & Sons, Inc., 1997) 358.

[23] Bellhouse, David, "Decoding Cardano's *Liber de Ludo Aleae*," *Historia Mathematica*. 32(2005). 185.

[24] *Ibid.*, 199.

clearly stated. When one thinks that he/she has figured out the question, it is frequently found that Cardano contradicts himself.[25]

The biggest complaint that modern mathematicians have about *Liber de Ludo Aleae* is that it is extraordinarily hard to understand. In his biography of Cardano, Ore gives us three main reasons as to why his writing is very difficult to understand: lack of modern mathematical symbols, lack of organization, and the simultaneous use of two different methods.[26]

Symbols

If one was to look at early mathematical texts compared to mathematical texts of today, there would be an enormous difference in the way the old text is written compared to the newer text. Cardano wrote his book in the early 16th century. This was long before most of the mathematical symbols that we use today had been created. Thus, there are not many straightforward and noticeable problems or equations in the book. Furthermore, most of the results he obtained were presented rhetorically as opposed to symbolically. With the lack the mathematical symbols, the reader is forced to infer all the general principles from the treatment of the few specific examples that he puts in the beginning of the book.[27]

Organization

Another problem that one may find in the book is that it does not have any reasonable form of organization. Cardano jumped back and forth between ethics, morals and probability theory. Since he initially created this book to serve as a gambler's

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[25] *Ibid.*, 180.
[26] Ore, 144-145.
[27] *Ibid.*, 145

manual, he was not very meticulous in his organization. In the book, Cardano placed his notes and his results together, without stating which was which. Furthermore, he did not correct his mistakes upon making them. After the reader establishes what Cardano was trying to show, the text is slightly easier to read. Cardano used a lot of trial and error. This is a good method; however, again he did not distinguish between his errors. Sometimes he simply fixed the error without alerting the reader. His haphazard editing causes much confusion.[28]

Methods

The last of the three main problems with his *Liber de Ludo Aleae* is that he used two completely different methods, sometimes even simultaneously. He used both the standard method and the ROTM method. One method works almost all the time while the other is only sometimes adequate. The standard method, which is the direct count of the various possible cases, almost never failed Cardano. However, when Cardano came upon a more difficult problem where the direct count of cases exceeded his knowledge, he resorted to the ROTM method. This method always gave an approximate answer, but never a concrete answer.[29]

Cardano knew that his results did not agree with each other and his observations. He could not give any reasons as to why they did not agree, thus this caused great confusion for those reading his book. It seemed like he resorted to the ROTM when he could find no other way to get an appropriate answer with the standard method.[30]

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[28] *Ibid.*, 145
[29] *Ibid.*, 145
[30] *Ibid.*, 145

Clearly, there are many downsides and negative feelings toward the book; however, if the reader views it as a gamblers' manual opposed to a mathematical text, there are advantages. First, it gives reasons for why one should gamble and how they should go about doing it in order to have fun instead of developing an addiction. There are also many red flags that Cardano pointed out so that the gambler is made aware of when someone is cheating or trying to rig the game.

As a mathematical text, *Liber de Ludo Aleae* only gives a basic understanding of the theory of probability. Cardano did not go into much depth due to the lack of knowledge on the theory of probability at the time.

Most people know of Cardano, not for his advancement in the theory of probability, but for his other achievements, such as his work on algebra. *Liber de Ludo Aleae* is almost always read by no one and disliked by all modern mathematicians. He did not live a wealthy life; however, he did in fact make significant contributions that improved the mathematical world. Although *Liber de Ludo Aleae* is not the most lucid work of all time, it provides adequate basics on how one could begin to understand the theory of probability. Furthermore, this book can help with the understanding of gambling, maybe not for the mathematics and probability aspects per se, but for the ethics and morals behind the game. *Liber De Ludo Aleae* can only be adequately appreciated when taken in the context of the 16th century and read with the intent of the author, Cardano, in mind.

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