**Geometry Playground Activity**
**Fitting Circles**

Tools you will be using: point, circle, distance, move point or space, fix point.

*The Ant queen wants to give a circular plot of land to each ant upon retirement. She sends Pi to find out if there is enough land to give away. Though she does not tell Pi the exact number, she does say that there are "myriads of retired ants that deserve a cozy new home." Pi takes his measuring tools and begins his search. Though he also does not know how big the plots of land need to be, he is reminded of the queen's use of the adjective "cozy" and decides that a plot of land with radius (approximately) 1 (foot) will do. He also needs to make sure that retired ants get their privacy: no two properties can share land (i.e. the circles cannot intersect).*

1. He begins in the Euclidean plane and quickly decides there is an abundance of land to be found. Why don't you follow him to the Inverted model and make sure he is right? If you want to make a circle bigger or smaller, Pi recommends that you fix its center and move the other point.

   What do you notice? Write down three observations you made about circles in the Inverted model.

2. Next he travels to the Hyperbolic plane. Here, Pi scratches his head in confusion. The land looks so small; he doesn't think you can fit many plots of land here. But he knows not to trust first impressions by now, so he takes out his measuring tools and begins.

   a. What do you think he discovers? Do you think there will be enough room? Go to at least two different models of the Hyperbolic plane before you give your answer, and then explain how you reached your conclusion.

   b. Let us for a moment focus on the Poincaré model only. You probably noticed that to the Euclidean eye, the circles seem to get smaller as you get closer to its boundary. In particular, construct a circle with radius 1 (or close to 1) and move the entire circle closer to the boundary of the model. What happens? Why do you think that is?

3. Next, Pi travels to the Spherical world. After only a few measurements, he decides that this will not be a good place for the retirement communities.

   How do you think he decided that? In particular, what is the largest circle you are able to construct? You can pick only one model to focus on (Pi thinks that the Sphere model might be easiest).

4. Having visited these three provinces, Pi travels back to the queen and gives her a report.

Which recommendation should he give to the queen? Which part of her land should she allot to the retired ants? Why? Give a mathematical explanation.